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FORCED OSCILLATIONS IN A HOMOGENEOUSLY FLUIDIZED BED

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The characteristics of the steady-state oscillations in a bed of finite height are considered; the frequency dependence of the amplitude is oscillatory, which enables one to identify discrete spectra of resonant and antiresonant frequencies.

One of the promising ways of accelerating transfer processes in fluidized beds is to superimpose an oscillation by means of pulsations in the pressure or flow rate of the fluidizing medium, or oscillations in the distribution grid, etc. In some cases, this simplifies the fluidization of finely divided materials, in which clumping is characteristic, and it also enables one to expand the existence limits for homogeneous fluidization. We therefore have to consider the distribution of the amplitude of the oscillations in the porosity, phase velocities, and so on over the volume of the layer and the relationship of these to the physical and other parameters of the system and to the external perturbation.

The problem has been considered on several occasions for unbounded beds in relation to the stability of the homogeneous fluidized state (see [1-5] and reviews in [6, 7]). These studies imply instability in small perturbations, and the stabilizing effect of the internal stresses in the dispersed phase are insufficient to provide stability at values of the parameters usually employed in fluidization [4, 5]. As the perturbations propagate, the nonlinear interactions between the perturbations differing in wavelength become important, which results in a generation of waves of considerable amplitude [8, 9], with a subsequent possible formation of bubbles and other discontinuities and the establishment of inhomogeneous fluidization.

To a considerable extent, these conclusions were drawn because no allowance was made for the finite time spent by a perturbation in the bed or the stabilizing effect of the upper boundary. This time is finite in a bed of finite height and sometimes is insufficient for the perturbation amplitude to increase substantially (particularly in the fluidization of small particles by liquids), while the upper boundary in principle can give rise to a system of reflected waves, which interfere with the initial ones [10, 11].

Here we consider the propagation of forced weak perturbations in a bounded layer of small particles fluidized by a gas. We neglect inertia, gravity, and viscous stresses in the gas, while the hydraulic resistance of the bed is considered a linear function of the infiltration speed. These assumptions simplify the expressions considerably but do not affect the essentials of the problem.

Linearized Equations. We consider the fluidized bed in a continuum approximation and write the equations for conservation of mass and momentum of the phases in the form

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{w}) = 0,\ \frac{\partial \rho}{\partial t} - \operatorname{div}(\varepsilon \mathbf{v}) = 0,\ d_1\rho\frac{d\mathbf{w}}{dt} = \mathbf{f} + d_1\rho\mathbf{g},\ -\nabla p - \mathbf{f} = 0,\ \varepsilon = 1 - \rho. \tag{1}$$

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For f we use a theory [12, 13] in which this force is represented as follows for a steady-state flow in a two-phase system with small particles:

$$\mathbf{f} = \rho\,[\beta K(\rho)(\mathbf{u} - \mathbf{v}) + d\mathbf{g}], \quad \mathbf{u} = \varepsilon\mathbf{v} + \rho\mathbf{w}, \quad d = \varepsilon d_0 + \rho d_1. \tag{2}$$

Here the first term describes the hydraulic resistance exerted by the particles with respect to the gas flow, while the second describes the effective upthrust (Archimedean force). A difference from most earlier studies [7] and certain unjustified recommendations based on a purely phenomenological approach [14] is that in (2) it is not necessary to introduce a term containing the pressure gradient; here the Archimedean force is determined by the density of the mixture as a whole, not simply by the gas density. Further, $\beta = 9\mu_0/2a^2$ for the flow around the individual particles at low Reynolds numbers; the monotonically increasing function $K(\rho)$ describes the effect of hindrance to the flow around the particles, and $K(0) = 1$. This function has been estimated [15] for moderately concentrated system: $K(\rho) = (1-5\rho/2)^{-1}$, and numerical calculations of the type of those of [16] are required to determine this for $\rho > 0.2$–0.3, so it may be more convenient to use one of the empirical relationships listed in the review of [6].

It can be shown from the method of [12, 13], and also is clear from general physical considerations, that allowance for the inertial force under nonstationary conditions leads to the replacement of **g** by **g**-d**w**/dt; further, $d_0 \ll d_1$, which enables us in particular to neglect the acceleration of the adjoint mass of gas, so instead of (2) we have

$$\mathbf{f} = d_1\rho\,[\varepsilon\alpha(\rho)(\mathbf{v} - \mathbf{w}) - \rho(\mathbf{g} - d\mathbf{w}/dt)], \quad \alpha(\rho) = (\beta/d_1)K(\rho). \tag{3}$$

We direct the x axis in the opposite sense to the **g** vector and give the solution of (1) describing the steady-state homogeneous condition in the layer:

$$v_0 = g/\alpha_0, \quad w_0 = 0, \quad p_0 = d_1\rho_0 g(h_0 - x), \quad \alpha_0 \equiv \alpha(\rho_0). \tag{4}$$

As pressure origin we take the pressure at the upper boundary of the bed $x = h_0$, while the x coordinate is reckoned from the distribution grid.

A perturbed state of the bed differing slightly from homogeneous is described by introducing quantities of the type $\varphi = \varphi_0 + \varphi'$, where by φ we understand any of the quantities ρ (or ε), p, v, and w. We have a system of linear equations for the prime variables from (1):

$$\begin{gathered}\frac{\partial\rho'}{\partial t} + \rho_0\frac{\partial w'}{\partial x} = 0, \quad \left(\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial x}\right)\rho' - \varepsilon_0\frac{\partial v'}{\partial x} = 0,\\ \frac{\partial w'}{\partial t} = \alpha_0(v' - w') + \alpha_0' v_0\rho', \quad \alpha_0' \equiv \left.\frac{d\alpha}{d\rho}\right|_{\rho=\rho_0},\\ \frac{\partial p'}{\partial x} = -d_1[\rho_0\alpha_0(v' - w') + (\alpha_0 + \rho_0\alpha_0')v_0\rho'].\end{gathered} \tag{5}$$

We consider a monochromatic wave with frequency ω and assume that $\varphi' = \Phi\exp(\lambda x + i\omega t)$ in accordance with the standard method. Then from (5) we get a system of linear algebraic equations for the amplitudes of the concentration, pressure, and velocity oscillations, whose characteristic equation has the roots

$$\lambda_1 = \lambda_2 = 0, \quad \lambda_3 = \lambda = \frac{\varepsilon_0\omega^2 - i\alpha_0\omega}{\rho_0(\alpha_0 + \varepsilon\alpha_0')v_0}. \tag{6}$$

The general solution to (5) is written as

$$\begin{gathered}\rho' = (R_1 + R_2 x + R_3 e^{\lambda x})e^{i\omega t}, \quad p' = (P_1 + P_2 x + P_3 e^{\lambda x})e^{i\omega t},\\ v' = (V_1 + V_2 x + V_3 e^{\lambda x})e^{i\omega t}, \quad w' = (W_1 + W_2 x + W_3 e^{\lambda x})e^{i\omega t},\end{gathered} \tag{7}$$

where it follows from (5) that

$$R_1 = R_2 = 0, \quad V_2 = W_2 = 0, \quad V_1 = \frac{i\omega + \alpha_0}{\alpha_0}W_1,$$

$$R_3 = -\frac{\rho_0\lambda}{i\omega}W_3, \quad V_3 = -\frac{\rho_0}{\varepsilon_0}\,\frac{i\omega + v_0\lambda}{i\omega}W_3,$$

$$P_2 = -id_1\rho_0\omega W_1,$$
$$P_3 = d_1\rho_0 \frac{i\alpha_0\omega + (\alpha_0 + \rho_0\varepsilon\alpha_0')\,v_0\lambda}{i\varepsilon_0\omega\lambda} W_3, \tag{8}$$

and λ is defined in (6). Equations (7) and (8) allow us to express the solution to (5) for a monochromatic wave completely apart from the three constants W_1, W_3, and P_1, which have to be determined from the boundary conditions.

The system allows only of bending or traveling waves propagating upwards with a velocity of the order of v_0, which is independent of frequency; the buildup increment is proportional to the square of the frequency. It is obvious that the system does not allow for waves reflected from the upper boundary of the type examined in [11].

Use of Boundary Conditions. We first consider the boundary conditions that must be imposed at the upper surface of the bed no matter what the situation at the lower boundary. In the present system of symbols, we have $h = h_0 + h' = h_0 + He^{i\omega t}$ for the height of the bed. Clearly, the particle velocity at the upper boundary must coincide with dh/dt and therefore we get the following kinematic condition from (7) and (8):

$$W_1 + W_3 \exp(\lambda h_0) = i\omega H. \tag{9}$$

If we neglect gas inertia, the pressure above a bed should be independent of the oscillations within it. Therefore, up to terms of the first order in small perturbations we get

$$P_1 + P_2 h_0 + P_3 \exp(\lambda h_0) = d_1\rho_0 g H \tag{10}$$

(here we have used the expression for the unperturbed pressure in (4)).

The expression for the amplitude H of the oscillations at the upper boundary is readily obtained from the condition for conservation of the suspended material in the bed, on which basis the integral of ρ with respect to dx with limits zero and h_0 should be $\rho_0 h_0$. Then from (7) and (8) we get with the former accuracy that

$$H = -\frac{1}{\rho_0\lambda}[\exp(\lambda h_0) - 1]\,R_3 = \frac{1}{i\omega}[\exp(\lambda h_0) - 1]\,W_3. \tag{11}$$

Equations (9)-(11) enable us to express two of the unknown constants in terms of one other. We take W_3 as the unknown quantity and get

$$W_1 = -W_3,\ P_1 = -\frac{d_1\rho_0}{i\varepsilon_0\omega\lambda}[\varepsilon_0(\alpha_0 v_0 - h_0\omega^2)\,\lambda + (i\alpha_0\omega + \rho_0(\alpha_0 + \varepsilon_0\alpha_0')\,v_0\lambda)\exp(\lambda h_0)]\,W_3. \tag{12}$$

The constant W_3 must be determined from an additional boundary condition, whose form is dependent on the details of the problem. Here we give boundary conditions corresponding to cases where the oscillations are set up by: 1) flow-rate pulsations, 2) pulsations in the gas pressure in the cavity under the distribution grid, and 3) grid vibrations. In view of the linearity of the problem, it is sufficient to consider only harmonic pulsations.

Let the gas flow through the grid in the first case be $q = q_0 + Qe^{i\omega t}$, where $q_0 = \varepsilon_0 v_0$ and Q is a given small quantity. On the basis of the above equations and the definition of the flow $q = \varepsilon v$ we get with the above accuracy that

$$-v_0 R_3 + \varepsilon_0 (V_1 + V_3) = Q. \tag{13}$$

In the second case let the pressure under the grid be $p^* = p_0^* + P^* e^{i\omega t}$, with $p_0^* = d_1\rho_0 g h_0 + kq_0$ in the unperturbed steady state (here we have used (4) and assumed that the hydraulic resistance of the grid is linearly related to the gas flow rate). In the perturbed state we again have $q = q_0 + Qe^{i\omega t}$, where $Qe^{i\omega t}$ is readily determined as a result from dividing the pressure drop at the grid by k. A simple calculation gives the boundary condition as

$$-v_0 R_3 + \varepsilon_0 (V_1 + V_3) = \frac{1}{k}(P^* - P_1 - P_3). \tag{14}$$

Finally, let the grid vibrate in such a way that the coordinate is $x = Ae^{i\omega t}$, where the amplitude A is sufficiently small for the motion not to affect the gas pressure under the grid, i.e., $p^* = p_0^*$; the pressure drop across the grid is clearly $k(q-i\omega Ae^{i\omega t})$, and we equate this to the difference of p_0^* and the instantaneous value of the pressure at the lower boundary of the bed to get

$$-v_0 R_3 + \varepsilon_0 (V_1 + V_3) = \frac{1}{k}(i\omega kA - P_1 - P_3). \tag{15}$$

Therefore, the mathematical formulations in the second and third cases are of the same type; the effects of grid vibration on the oscillatory process may thus be considered by examining the system with an immobile grid with pressure pulsations underneath with the complex amplitude $P^* = i\omega kA$.

It will be incorrect to formulate the boundary conditions for the third case by equating the velocity of the particles at the lower boundary to the velocity of the grid $i\omega Ae^{i\omega t}$; in fact, the rising grid results in a densely packed layer of particles above it, while a descending grid produces a gas layer free from particles; in both cases the situation is clearly not described by (5).

Using (8) and (12) we get the following expressions for W_3 in the case of flow-rate pulsations [condition (13)]:

$$W_3 = -Q\left(1 + \frac{i\varepsilon_0\omega}{\alpha_0}\right)^{-1}, \tag{16}$$

and for pulsations in the pressure [condition (14)]:

$$W_3 = -P^*\left\{k\left(1 + \frac{i\varepsilon_0\omega}{\alpha_0}\right) + \frac{d_1\rho_0}{i\varepsilon_0\omega\lambda}\left[\varepsilon_0(\alpha_0 v_0 - h_0\omega)\lambda + (i\alpha_0\omega + (\alpha_0 + \varepsilon_0\alpha_0')v_0\lambda)(\exp(\lambda h_0) - 1)\right]\right\}^{-1}. \tag{17}$$

In the case of grid vibrations [condition (15)], (17) applies, but with P^* replaced by $i\omega kA$. For $k \to \infty$, this formula becomes (16), where $i\omega A$ appears instead of Q.

These equations completely close the solution to (7) for these problems. Some simple but somewhat cumbersome calculations enable us to examine the dependence of the amplitudes and phase angles of the oscillations of the various quantities at various levels in the bed in relation to the various physical parameters. This is carried through here for the first case, where the oscillations are excited by flow-rate pulsations.

Example. Excitation of oscillations by flow-rate fluctuations. We introduce the dimensionless parameters

$$\nu = \frac{\omega}{\alpha_0},\ \varkappa = \frac{\alpha_0'}{\alpha_0},\ \sigma = \frac{\alpha_0 h_0}{v_0} = \frac{\alpha_0^2 h_0}{g},\ \gamma = \frac{\sigma}{\rho_0(1+\varepsilon_0\varkappa)}. \tag{18}$$

The values of $\varkappa$ and γ are dependent on the form of $K(\rho)$. If we use Ergan's formula for small particles to determine this, then $\alpha \sim \varepsilon^{-2}$, i.e., $\varkappa = 2/\varepsilon_0$ and $\gamma = \sigma/3\rho_0$, and for $a \sim 10^{-2}$ cm the characteristic values are $\alpha_0 \sim 10^2$-10^3 sec^{-1} and $\sigma \sim \gamma \sim 10^2$-$10^4$.

The simplest form is taken by the wave for the volume concentration of the dispersed phase (or porosity). From (7), (8), and (16) we have

$$\rho' = -\frac{\gamma\rho_0}{\alpha_0 h_0} Q \exp[\varepsilon_0\gamma\nu^2\xi + i(\omega t - \gamma\nu\xi)],\ \xi = \frac{x}{h_0}, \tag{19}$$

which is a wave propagating upwards with an exponentially increasing amplitude. The linear theory clearly applies for

$$Q\exp(\varepsilon_0\gamma\nu^2) \ll \alpha_0 h_0/\gamma \sim g/\alpha_0, \tag{20}$$

which imposes a constraint on the amplitude of the flow-rate pulsations and the increment in the amplitude. The latter is small for a fairly wide class of systems, so the amplitude increases linearly with height (a quantity analogous to this increment was used in [11] as a small parameter). Condition (20) thus leads to the inequality

$$Q \ll g/\alpha_0,\ \nu \lesssim \nu^{(1)} = 1/\sqrt{\gamma} \sim (1/\alpha_0)\sqrt{g/h_0}. \tag{21}$$

It is not in fact correct to simulate the phases in a fluidized bed as two mutually penetrating and interacting continuous media, which leads to (1), if the linear scale for substantial variation in the hydrodynamic variables substantially exceeds the particle size. This imposes a further constraint on the dimensionless pulsation frequency:

$$\nu \ll \nu^{(2)} = (1/\gamma)(h_0/a). \tag{22}$$

If this inequality is violated, i.e., $\nu \gtrsim \nu^{(2)}$, one cannot use (5), which follows from (1), to describe the oscillations. In that case it is necessary to consider the interaction of the individual particles with a substantially inhomogeneous gas flow. The strength of this interaction and the related energy dissipation increase sharply as the linear scale of the flow is reduced, i.e., in the present case with increase in the pulsation frequency, as occurs in the interaction of particles with small-scale turbulent eddies [17]. These high-frequency oscillations should die out rapidly as the height above the grid increases, and therefore they need not be examined. For $h_0 \sim 10$ cm, $a \sim 10^{-2}$ cm and $\gamma \sim 10^3$ we have $\nu^{(1)} \sim 10^{-1.5}$, $\nu^{(2)} \sim 1$, which corresponds to the frequencies $\omega^{(1)} \sim 10$ Hz, $\omega^{(2)} \sim 10^{2.5}$ Hz (we have assumed that $\alpha_0 \sim 10^{2.5}$ sec^{-1}).

From (11) and (16) we have for the complex oscillation amplitude of the upper boundary that

$$H = \frac{iQ}{\omega(1 + i\varepsilon_0\nu)}[\exp(\gamma\nu(\varepsilon_0\nu - i)) - 1]. \tag{23}$$

The real amplitude

$$F_h = \frac{|H|}{Q/\alpha_0} = \frac{1}{\nu(1+\varepsilon_0^2\nu^2)^{1/2}}[\exp(2\varepsilon_0\gamma\nu^2) + 1 - 2\exp(\varepsilon_0\gamma\nu^2)\cos(\gamma\nu)]^{1/2} \tag{24}$$

is an oscillatory function of the parameter $\gamma\nu$, which is modulated by the functions

$$F_h^{+,-} = \frac{1}{\nu(1+\varepsilon_0^2\nu^2)^{1/2}}[\exp(\varepsilon_0\gamma\nu^2) \pm 1]. \tag{25}$$

Figure 1 shows the dependence on the dimensionless frequency of the quantities of (24) and (25); F_h is clearly an oscillatory function of the dimensionless frequency whose amplitude decreases as the frequency rises. There are discrete spectra of resonant and antiresonant frequencies, which correspond to comparatively flat maxima and sharper minima. In the frequency range of main interest, the relationship differs little from harmonic, i.e., that obtained with $\exp(\varepsilon_0\gamma\nu^2) \approx 1$; in particular, we have approximately what follows for the frequencies providing the maxima and minima in the amplitudes at the upper boundary:

$$\nu_{\max} \approx (2n+1)\pi/\gamma, \quad \nu_{\min} \approx 2n\pi/\gamma, \quad n = 0, 1, \ldots \tag{26}$$

For the real amplitude of oscillation of the particle velocity in the bed we have

$$F_w = \frac{|W|}{Q} = \frac{1}{(1+\varepsilon_0^2\nu^2)^{1/2}}[\exp(2\varepsilon_0\gamma\nu^2\xi) + 1 - 2\exp(\varepsilon_0\gamma\nu^2\xi)\cos(\gamma\nu\xi)]^{1/2}. \tag{27}$$

Figure 2 shows the ν dependence for this quantity for various ξ, i.e., for various levels within the bed; relationships differ little from the corresponding harmonic ones. At each level there are discrete sets of frequencies for which the amplitudes are maximal and minimal. The intervals between the different $\nu_{\max}$ and $\nu_{\min}$ decrease in inverse proportion to ξ as the height above the bed increases. The ξ dependence of F_w for various ν is analogous to that of the relationship shown in Fig. 2.

It is also simple to consider the oscillations in phase and pressure. For ν sufficiently small (such that one can assume $\exp(\varepsilon_0\gamma\nu^2) \approx 1$) we have from the above that

$$F_v = \frac{|V|}{Q} \approx 1 + \left(\frac{\rho_0}{\varepsilon_0}\right)^2\left(1 - \frac{\gamma}{\sigma}\right)^2 + 2\frac{\rho_0}{\varepsilon_0}\left(1 - \frac{\gamma}{\sigma}\right)\cos(\gamma\nu\xi), \tag{28}$$

and also that

$$F_p = \frac{|P|}{d_1\rho_0\alpha_0 Q h_0} \approx \frac{\sqrt{2}}{\sigma\nu}[1 - \xi(1-\xi)(1-\cos(\gamma\nu)) - \xi\cos(\gamma\nu(1-\xi)) - (1-\xi)\cos(\gamma\nu\xi)]. \tag{29}$$

In (29), we have used the following quantity instead of the dimensionless coordinate $\xi = x/h_0$ used in the other equations:

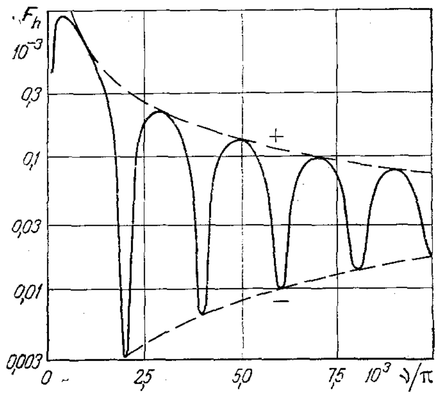

Fig. 1

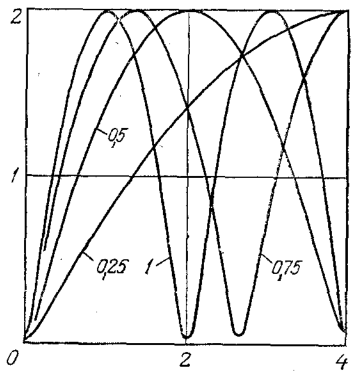

Fig. 2

Fig. 1. Dependence of the dimensionless real amplitude of the oscillations of the upper boundary on the dimensionless frequency for $\gamma = 10^3$ and $\varepsilon_0 = 0.5$; the broken lines show the modulating function.

Fig. 2. Dimensionless real oscillation amplitude in particle velocity at various levels in the bed (the numbers at the curves give the values of $\xi = x/h_0$) as functions of the dimensionless frequency.

$$\xi = \frac{x}{h} \approx \frac{x}{h_0}\left(1 - \frac{H}{h_0} e^{i\omega t}\right), \tag{30}$$

which enables us to give the formula a form symmetrical with respect to the middle level $\xi = 0.5$.

The relationships of F_V and F_p to ξ and ν are very similar to the corresponding ones for F_W. There are also characteristic resonant and antiresonant frequencies, which coincide with those for the velocity fluctuations. The bed contains systems of standing and traveling waves that are simulated by certain standing waves in which the positions of the nodes and antinodes are determined by $\gamma\nu$.

The mechanism whereby these standing waves are produced is quite different from that indicated by the analysis of [11], where the upper boundary was considered as a surface capable of reflecting incident waves. The above theory indicates that reflected waves do not occur at all in the system, and the stabilizing action of the upper boundary is simply due to degeneration of the pressure fluctuations there. The latter can be seen for example from (29).

It has been shown by experiment [10] that there is a frequency at which the pressure fluctuations are the strongest at a certain height above the grid. However, the theory of [10] proposed to explain this phenomenon was based on representing the fluidized bed as a homogeneous elastic medium, which is inadequate. Also, there is not a single frequency providing the maximum amplitude in the gas pressure and in the other hydrodynamic quantities but an infinite set of frequencies (here it must be remembered that the theory developed here ceases to be correct at high frequencies).

These results have applied significance. In principle, it is possible to select the frequency of the external force in fluctuations and the parameters of the bed to provide the maximum pulsation intensity at different parts of the bed with minimum fluctuations in the height. Therefore, future work in this area would appear to be very promising, particularly detailed study of the oscillations produced by other means (e.g., by pressure oscillation or grid vibrations), with transfer of the results to beds fluidized by liquids (which as a rule are in fact homogeneous), together with experimental refinement of the theoretical relationships between the optimum frequencies for accelerating the transfer processes and the physical and other parameters of the bed.

NOTATION

A, grid vibration amplitude; a, particle radius; d_0, d_1, d, gas, particle and mixture densities; F, dimensionless actual fluctuation amplitude; **f**, interphase interaction force; **g**, gravitational acceleration; H, amplitude of the upper bed boundary fluctuations; h, bed height; K, function for constrained flow around particles; k, grid drag coefficient; p, pressure; q, gas flow rate; t, time; **u**, **v**, **w**, mixture, gas and particle velocities; x, longitudinal coordinate; P_i, R_i, V_i, W_i, constants in (7); α, β, quantities introduced into (3);

ε, porosity; λ, root of the characteristic equation; μ_0, gas viscosity; ν, dimensionless frequency; ξ, dimensionless coordinate; ρ, volume particle concentration; ω, frequency; γ, ϰ, σ, parameters introduced into (18). Indices: 0, undisturbed state of the bed; ', pulsations of hydrodynamic quantities and differentiation of α with respect to ρ; *, state under the gas-distributing grid.

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METHOD OF INTERPOLATING DATA IN DETERMINING THE RHEOLOGICAL PARAMETERS OF A LIQUID

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UDC 541.148:66.011

The least-squares method is applied to determine the parameters in the rheological equation of state for the liquid over wide ranges in the velocity gradient and tangential stress.

The rheological parameters of liquids have major effects on the hydromechanical, thermal, and mass-transfer processes [1], and therefore correct determination of the rheological equation of state for a liquid is a basic problem in rheology [2, 3].

The ranges in strain rate and stress for a given object frequently constitute 4-6 orders of magnitude, so mathematical description of experimental values usually involves piecewise approximation for individual ranges in

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